# An Economic Analysis of the Tourism Contracts: Allotment and Free Sale*

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**Abstract**

In this paper we present an economic analysis of the typical incomplete tourism contracts regulating the business relationship between the Tour Operator and the Hotel: the Allotment and the Free Sale contracts.

Starting from an "optimal contract design" approach, we study the analytical solution of the optimization choices under uncertainty of the these firms. Two questions are specifically dealt: (i) how the Tour Operator and the Hotel choose the optimal contract terms (quantity and price); (ii) which are the "optimal risk sharing" rule (Allotment contract) and the "optimal risk allocation" rule (Free Sale contract).

Finally, we compare the two contracts in terms of their efficiency and Pareto optimality.

## 1 Introduction

In the tourism sector it is very difficult to observe "complete contracts". The typical tourism incomplete contracts regulating the business relationship between the Tour Operator and the Hotel are the Free Sale and the Allotment contracts. In this paper we analyze these incomplete contracts into a microeconomic perspective and within an "optimal contract design" approach[1]. In particular through "optimal contract design" and "incomplete contract" approaches we analyze the design and efficiency consequences of imperfections resulting from contractual incompleteness[2].

We shall compare the observed stylized facts - the actual business relationships and the contracts - with the results from the present models because we

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[1] For more details on "optimal contract design", see Shavell S. (1984), Hart O.D. and Holmstrom B. (1987), Holmstrom B. and Tirole J. (1989), Werin L. and Wijkander H. (1992), Tirole J. (1994), Masten S.E. (1988) and (1999).

[2] See Holmstrom B. and Tirole J. (1989), Tirole J. (1994), Masten S.E. (1999).

want to verify if the economic analysis applied to contracts can be used to look for solutions to some of the typical tourism economic problems. We also want to check if tourism economics is capable to provide conceptual tools that can be applied to a more general framework. Two questions are specifically dealt within our setting: (i) how the Tour Operator and the Hotel choose the optimal contract terms (quantity and price), given the initial conditions; (ii) which are the "optimal risk sharing" rule (Allotment contract) and the "optimal risk allocation" rule (Free Sale contract).

The paper is structured as follows. In section 2 we present the main stylized facts of tourism sector and the assumptions for our model. In section 3 we show the Free Sale and the Allotment contracts within a framework of "law and economics". In sections 4 and 5 we set up the models of the two contracts (Allotment and Free Sale) and we obtain their analytical solutions. A comparative analysis between the two models in terms of "optimal contract design" concludes the paper.

## 2 Stylized Facts and Assumptions

There are some stylized facts in the tourist sector that are very remarkable when the contractual relationships between the Tour Operator and the Hotel have to be analyzed.

One of the peculiarity of the tourism sector is that market demand is subject to seasonality and uncertainty. The tourism market demand is stochastic because it depends on stochastic and exogenous variables, e.g. natural resources, institutional aspects, social and cultural elements, economic and political conditions, psychological influence. Further uncertainty is due to the time that passes from the "purchase" (booking) to the "consumption" of a tourist-industry product (time consuming). Additionally, goods in this industry are typical experience goods and the consumption is time intensive. For these reasons in this paper we assume that the demand, the reservation price, the frequency and the duration of holidays are all stochastic variables.

Another stylized fact in the tourism sector is that the tourist product is a non-storable good[3]. In fact, the final product supplied to the customer is not a physical good, but an immaterial service-good. For this reason - and given the seasonality and uncertainty of the market demand - the Tour Operator and the Hotel are subject to many economic risks in carrying out their activity[4].

Since the Tour Operator is exposed to an economic risk due to the risk of market demand changes and the Hotel is subject to the economic risk due to the possibility that some rooms remain empty (unsold), the Free Sale and the

[3] For example an empty room in an Hotel is a service "produced" but not sold.

[4] It should be stressed that we are speaking of "economic risks" and not of "technical risks", related for example to the safety of person, transport or baggages. In these latter cases there is nothing different from the normal insurance contracts.

Allotment contracts can be of help in managing these risks[5]. Therefore in this paper we assume that the Tour Operator operates only in the future market, while the Hotel can sell its services (free rooms) both to the Tour Operator on the future market and directly to the tourists on the spot market[6].

Another important feature of the tourism market is the existence of a high degree of monopoly power. In fact, both the Tour Operator and the Hotel act in market structures which can be described by monopolistic competition or oligopoly models. The tourist services of the Hotels are extremely differentiated with respect to the location, to the typology of customers, to the typology and quality of services supplied, to the facility dimension and to the ownership and it is therefore reasonable to assume a high degree of monopoly power, as we do in our model. Since the main service supplied by the Tour Operator in the "package tour" is the Hotel accommodation, the analogous market structure can be assumed for the Tour Operator. For this reason it is reasonable to assume that the method used by the Tour Operator to charge the supply price (or "posted price") is based on the mark-up percentage[7].

Another feature of the tourism market is that the "tourist facility"[8] is characterized by a certain degree of rigidity. This rigidity means that the dimension of the facility can be changed by the firms only in the long run through investment decisions. For the purpose of this research, it is sufficient to analyze the firm decisions only in the short run, without taking into account fixed costs imputation and investment decisions[9].

Another stylized fact observable in the tourism sector is that the consumer-tourists usually buy in advance the tourism services on the future market only from the Tour Operator (or the Travel Agency), while they buy them directly from the Hotel only on the spot market[10]. For this reason we assume in our model that the future market of tourism services is opened only for the Tour Operator.

These issues reflect five important assumptions of our model, as we shall formally present in Sections 4 and 5: (i) a monopoly market, both for the

[5] Another way to manage the risk is the "overbooking" technique. See Candela G. and Figini P. (forthcoming).

[6] In fact, the available rooms can be sold by the Hotel either (i) on the future market, i.e. to the Tour Operator through the Free Sale or the Allotment contract (indirect distribution), or (ii) on the spot market, i.e. directly to the consumers-tourists (direct distribution). In case the quantity of rooms booked in advance by the Tour Operator is not sufficient to satisfy the market demand, then the Hotel has the possibility to sell these further rooms demanded by the market directly to the tourists at a higher spot market price.

[7] In our model we assume this percentage as an exogenous parameter.

[8] For the Hotel the "tourist facility" consists of the number of rooms available (capacity), while for the Tour Operator of the maximum number of persons for which the "package tour" is planned.

[9] As we will see, we will formalize in our model this rigidity of the Hotel supply through the introduction of a capacity constraint.

[10] In fact, since the Tour Operator signs in advance the agreements with its suppliers, it is then capable to manage accommodation and transportation services with ease and saving of time and money, and moreover it can guarantee a certain level of safety, quality and price for the holiday.

Tour Operator and the Hotel[11]; (ii) a stochastic tourism market demand and thereafter a framework of choices under uncertainty; (iii) tourist products are non-storable services; (iv) a short run temporal horizon for firm decisions and accordingly a capacity constraint for the Hotel; (v) the future market is closed to the consumers-tourists.

# 3 The Free Sale and the Allotment contracts

The Free Sale and the Allotment contracts are accommodation contracts, in which the exchange consists of "cash for rooms", where the Hotel rooms are filled-in at a later stage: they are therefore contingent and future contracts, in which the main problem is to overcome the negative effects of uncertainty caused by deferred exchanges, that could limit the realization of the exchange[12]. Therefore, such contracts are proposed by the Tour Operator to the Hotel in order to share (the Allotment) or to cover (the Free Sale) the economic risk brought about by the possibility of unsold rooms due to a potential low market demand. The exchange and the level of the two main variables involved, i.e. price and quantity of booked rooms, depend therefore on the following aspects: the propensity, neutrality or aversion towards the risk of the two agents, the probability they assign to the different possible stochastic events (high or low market demand) and their bargaining power.

In particular, the Free Sale contract can be regarded as a sort of insurance contract regulating a "risk-transfer" transaction, through which the Tour Operator grants against the economic risk of unsold rooms (insurance level) by booking in advance a fixed number of rooms (contractual quantity). The Hotel then gives in return a discount (insurance premium) on booking price (contractual price). Through this contract the Tour Operator purchases the agreed number of rooms (accommodations) related to a certain limited period of time and commits itself to pay the entire agreed price before the actual demand is known, i.e. independently of the future actual rooms occupancy. For this reason this contract is known in the literature as a "fixed quantity commitment" contract[13] and the reservation in advance of a fixed number of rooms, leaves the Tour Operator with a high risk due to the market demand uncertainty.

The Allotment contract is instead a typical "risk-sharing" agreement, which represents a mechanism of sharing the risk between both parties and is known

[11] It should be stressed that most of the phenomena analyzed in this paper could be derived even in the presence of competitors as long as the firm retained some market power. See Tirole J. (1988).

[12] See Masten S.E. (1999), page 29: "Sophisticated but boundedly rational transactors will omit contingencies when the costs of anticipating, devising optimal responses to, and drafting provisions for improbable events outweigh the expected gains in efficiency from doing so. Departures from the Arrow-Debreu ideal may thus arise in incomplete contract theory from failures of the contracting parties to foresee and provide for contingencies in formulating their agreement, instead of or in addition to the inability of courts to verify performance".

[13] See R. Anupindi and Y. Bassok (1998), and V.F. Araman, J. Kleinknecht and R. Akella (2003).

in the literature as a "quantity flexibility" contract[14]. In fact, in this case a certain number of rooms (contractual quantity) is available for sale by the Tour Operator for a certain limited period of time, and the Tour Operator commits itself to confirm or not to the Hotel the actual rooms occupancy within an agreed deadline (the "release date"). The Tour Operator pays for this right to confirm or not the actual rooms occupancy, in the form of an "option exercise price" equal to a percentage of the accommodation value (contractual price). The Tour Operator exercises this option (called "call option") at the release date (European call) or within the release date (American call) by informing the Hotel if it intends to confirm or to release the rooms. If instead the confirmation of the rooms occupancy is not exercised within the agreed deadline or in accordance with the contractual clauses, the Hotel can sell the empty rooms directly to the tourists on the spot market.

From the point of view of the economic theory of contracts, we can therefore affirm that the Free Sale and the Allotment contracts belong in the same time to the following typologies of contracts: (i) contingent contracts; (ii) future contracts. The main difference between them is instead that the Free Sale contract is an insurance contract (or a risk-transfer transaction), whose function is to shift the economic risk to the least risk-averse transactor or to the low-cost risk bearer, while the Allotment contract is an option contract (or a risk-sharing agreement), whose function is to share the economic risk between the two risk-averse transactors[15].

# 4 The Allotment Model

From the "stylized facts" of the tourism sector and the economic features of the two accommodation contracts described in the previous Sections, we derive the economic insights that allow us to build our models of the two contracts.

Let us present some common characteristics of the two models. For the Hotel as well as for the Tour Operator, the risk is increased by the seasonality and uncertainty of the market demand. It is therefore very important for these firms to foresee the market demand, in order to maximize firm profit. The two economic decisions to be taken are: (i) the number of rooms (for the Hotel) and thereby the number of "package tours" (for the Tour Operator) to be sold on the market; (ii) how to charge the selling price.

Therefore in our models the two decision variables are, respectively, the contractual quantity (number of rooms booked in advance by the Tour Operator within the Hotel capacity constraint) and the contractual price (purchase price for the Tour Operator which includes a discount given by the Hotel or an option price paid to the Hotel with respect to the spot market price). On the contrary, we shall assume as exogenous parameters the other economic variables involved

[14] See C. Li and P. Kouvelis (1999), A. Tsay (1999) and D. Barnes-Schuster, Y.Bassok and R.Anupindi (2000).

[15] See Cheung Steven N.S. (1969) and Stiglitz J. (1974).

(in particular, the Tour Operator mark-up and the Hotel production costs).

In Section 2 we have presented the topic assumptions of a seasonal and uncertain market demand and of a monopoly market both for the Tour Operator and for the Hotel[16]. Now we add the further assumption that the two firms have symmetric information such that they can have the same expectations about the future market demand[17].

Moreover the choice of the optimal decision under conditions of uncertainty depends substantially on the attitude towards the risk. Therefore the basic assumption that justifies the existence of the Allotment contract is that both the Tour Operator and the Hotel are risk-averse agents. In fact, the occurrence of two risk-averse agents is a necessary condition for a risk-sharing agreement, but not a sufficient condition for a Pareto optimal contract in a framework of "choices under uncertainty".

Usually the different "states of nature", on which the stochastic demand is conditioned, represent a continuous set of events but here, for simplicity, we consider only a discrete case. Let us therefore assume the states of nature are only two: $D^L$ (low market demand) and $D^H$ (high market demand)[18] and the corresponding probabilities assigned both by the Tour Operator and the Hotel are equal to $\alpha^L$ and $\alpha^H$. This implies that the tourism market demand is a random variable:

$$D = \left[D^L; D^H\right] \text{ with probabilities } \alpha^L \text{ and } \alpha^H \tag{1}$$

where $\alpha^L + \alpha^H = 1$ (therefore $\alpha^L = 1 - \alpha^H$).

We can specify the Allotment contract clauses in the following way. The contractual price is different depending on the choice of the Tour Operator to confirm - and therefore to buy - or to release the rooms. In these two cases the Tour Operator will pay: $P^a = \bar{P} + O(E)$ for confirmed rooms and $P^a = O(E)$ for released rooms or, which is the same, $P^a = \bar{P}$ for confirmed rooms and $P^a = O(E)$ for all booked rooms, where $P^a$ is the "contractual price"[19], while $\bar{P}$ represents the given price on the spot market, $O(E)$ is the option price paid by the Tour Operator to the Hotel, and $E$ is the Hotel supply, i.e. the "contractual quantity" (rooms booked in advance).

We assume the following capacity constraint for the Hotel[20]: $E \leq \bar{K}$where

[16] The market is therefore composed of two monopolistic tourist firms - the Tour Operator and the Hotel - and the consumers-tourists.

[17] This is a reasonable assumption since they are both in the position to gain access to the same information regarding the future behavior of the market demand.

[18] Note that a stochastic market demand could be introduced in the model by assuming "price uncertainty", instead of "quantity uncertainty" like we have done here. The case with "price uncertainty" is anyway symmetric with respect to our model, while the more complex case in which both uncertainties are assumed and analyzed simultaneously could represent an interesting implementation of this model.

[19] To ensure the economic significance of the price $P^a$, we introduce the following positivity constraint (positive price condition): $P^a = \bar{P} + O(E) \geq 0$ and therefore $\bar{P} \geq -O(E)$.

[20] We recall from Section 2 that the assumption of a short run temporal horizon for firm decisions implies the introduction of a capacity constraint for the Hotel. Moreover, it implies the choice of not taking into account fixed costs imputation and investment decisions.

$\bar{K}$ is the maximum number of rooms[21] that the Hotel can supply in the short run. Therefore the available rooms can be sold by the Hotel: either on the future market, i.e. to the Tour Operator through the Allotment contract (indirect distribution) or on the spot market, i.e. directly to the consumers-tourists (direct distribution).

In this way the Tour Operator and the Hotel - which we assume to be both risk-averse - share between themselves the risk of unsold rooms, through the mechanism of an option price $O(E)$ paid by the Tour Operator to the Hotel. In case the rooms $E$ booked in advance are partially released by the Tour Operator or on the contrary they are all confirmed but anyway not sufficient to satisfy the market demand, then the Hotel has the possibility to sell these further rooms directly to the tourists at the higher spot market price $\bar{P}$.

In terms of the Allotment contract clauses, we observe from the contractual stylized facts that the option price $O(E)$ paid by the Tour Operator for the right to confirm or not the actual rooms occupancy - which in our model is the endogenous component of the "contractual price" $P^a$ - is equal to a percentage of the accommodation value. We define this unit option price as a constant function: $O(E) = o$ where $0 \leq o \leq 1$ is a decision variable. The two decision variables of the Allotment model, with respect to which we want to find out an optimal decision rule, are: $E$, number of rooms booked in advance by the Tour Operator (contractual quantity) and $o$, unit option price paid by the Tour Operator to the Hotel (implicit contractual price).

Once the Tour Operator has decided to confirm the rooms, it will include them in the "package tours" and it will sell them (usually through the intermediation of a Travel Agency) at the future price (called "posted price" or "brochure price"): $P^s = \bar{P}(1+m)$ where $m > 0$ is the mark-up percentage applied by the Tour Operator, that we assume in our model as an exogenous parameter.

For simplicity we normalize the spot market price $\bar{P} \equiv 1$, so that the contractual prices become[22]: $P^a = 1 + o$ for confirmed rooms and $P^a = o$ for released rooms (or also $P^a = 1$ for confirmed rooms and $P^a = o$ for all booked rooms), and $P^s = 1 + m$.

It must be stressed that a "positive profit" condition for the Tour Operator - which is the natural consequence of the assumption of a monopoly market - implies that $P^s > P^a$, condition respected if $m > o$. In this way, we are also assuming that the Tour Operator is applying a "fixed price" strategy: its products ("package tours") are sold at a unique price that does not change in the period of time considered[23]. Similarly we can assume a "positive profit"

[21] For simplicity, we assume that all the rooms are standard rooms, for example double rooms.

[22] As previously stressed, the economic significance of the price $P^a$ implies the following positivity constraint: $P^a = 1 + o \geq 0$, and therefore $1 \geq -o$, always true by definition. The positivity of the price $P^s$ is instead implicit in the assumption $m > 0$.

[23] On the basis of this "fixed price" strategy, the selling prices could change only in a different season or for extraordinary reasons. This is actually one of the price strategies that are most used by the touristic firms, while other possible price strategies are: changing

condition also for the Hotel, that implies that $P^a > c$ (where $c > 0$ is the Hotel unit production cost[24]) and therefore, after price normalization, that $o > c$.

To simplify the analysis, we assume now the following condition on the quantity variable:

$$D^L \leq E \leq D^H \tag{2}$$

In fact, since the Tour Operator knows that market demand will be $D^L$ or $D^H$, it would not be economically rational to book: (i) less than $D^L$ rooms ($E < D^L$), otherwise the Tour Operator would certainly loose favorable opportunities, for an amount equal to $P^s\left(D^L - E\right)$; (ii) more than $D^H$ rooms ($E > D^H$), otherwise the Tour Operator would have a certain loss due to the option price to be paid for the Allotment contracts in excess (released rooms), for an amount equal to $O(E)\left(E - D^H\right)$.

The basic assumption of a stochastic market demand implies that the sales of the Tour Operator are stochastic too[25]. We therefore assume that the Tour Operator sales $S$ are a random variable conditional on observed demand, according to the so called "short-side rule"[26]: $S = \min[D, E]$ which is the "min" condition in the canonical disequilibrium model when the observed amount of transaction is equal to the minimum between the supply and the demand. The "short-side rule" assumption is consistent to the fact that the fixed selling price $P^s$ ("fixed price" strategy) is higher than the booking price $P^a$ ("positive profit" condition).

From these assumptions it follows that for the Tour Operator it is profitable to sell (to confirm) every room demanded by the market, within booked rooms, otherwise: (i) if $D = D^L$, by selling (confirming) $S^L < D^L$ the Tour Operator would have an opportunity cost equal to $P^s\left(D^L - S^L\right)$ with probability $\alpha^L$; (ii) if $D = D^H$, by selling (confirming) $S^H < E$ the Tour Operator would have a certain loss due to the option price to be paid for the released rooms, equal to $O(E)\left(E - S^H\right)$ with probability $\alpha^H$.

Another way to define the "short-side rule", is to express the Tour Operator sales as the following random variable:

$$S = \begin{cases} S^L = D^L \text{ with probability } \alpha^L = 1 - \alpha^H \\ S^H = E \text{ with probability } \alpha^H \end{cases} \tag{3}$$

To summarize, the model has the following assumptions: (i) a market composed of two monopolistic firms - the Tour Operator and the Hotel - and of

price depending on the quality of the service (vertical differentiation) or on the different characteristics of demand segments (price discrimination).

[24] We shall introduce later on the parameter "Hotel unit production cost" when we shall define the Hotel utility function over its profit, where the Hotel profit is equal to $\Pi_h = P^a S - cE$ and $c$ is the Hotel unit production cost as long as its capacity is not satiated.

[25] We assume that the rooms sold by the Tour Operator are identically equal to the rooms confirmed within the release date.

[26] According to the "short-side rule", or "Hahn-Negishi rule", the short side of the market determines the transaction quantity. See Hahn F. H. and Negishi T. (1962), Quandt R. (1988) and Fisher F.M. (1983).

consumers-tourists; (ii) a stochastic tourism demand; (iii) the form of an Allotment contract is analogous to an option contract; (iv) risk-aversion of the two transactors; (v) exogenous market parameters (spot market price, Tour Operator mark-up, Hotel unit cost). Through this model we want to find the optimal contract in terms of: (i) rooms $E$ booked in advance by the Tour Operator; (ii) option price $o$ paid to the Hotel.

To model the Allotment contract, we start from the definition of the utility functions of the Tour Operator and the Hotel and we end up with the corresponding optimization problem. Since the Tour Operator is an expected utility maximizer with a Bernoulli utility function over its profit[27], we can define its utility function as[28]: $U_{TO}(E,o) = Ex\left[u_{TO}\left(\Pi_{TO}(E,o)\right)\right]$ where $Ex$ stands for "expected value" and $\Pi_{TO}$ is the Tour Operator profit, which we define as: $\Pi_{TO}(E,o) = P^s S - P^a E$.

The Tour Operator utility becomes:

$$\begin{aligned} U_{TO}(E,o) &= \alpha^L u_{TO}\left[\bar{P}(1+m)S^L - \bar{P}S^L - oE\right] + \\ &\quad + \alpha^H u_{TO}\left[\bar{P}(1+m)S^H - \bar{P}S^H - oE\right] \end{aligned} \tag{4}$$

and applying the "short-side rule" (3) and taking into account that $\alpha^L = 1 - \alpha^H$, we have the final result[29]:

$$U_{TO}(E,o) = \left(1-\alpha^H\right) u_{TO}\left[mD^L - oE\right] + \alpha^H u_{TO}\left[mE - oE\right] \tag{5}$$

Since the Tour Operator maximizes its expected profit subject to the Hotel participation constraint, we define the Hotel utility function over its profit as follows[30]: $U_h(E,o) = Ex\left[u_h\left(\Pi_h(E,o)\right)\right]$.

We define the Hotel profit as: $\Pi_h(E,o) = P^a S - cE$, where $c > 0$ is the Hotel unit production cost[31]. This profit takes a different form depending on the actual market demand, because in case of low market demand $(D = D^L)$ it becomes $\Pi_h^L(E,o) = P^a S^L - cE$, while in case of high market demand $(D = D^H)$ it becomes $\Pi_h^H(E,o) = P^a S^H - cE + \left(\bar{P} - c\right)\left(D^H - S^H\right)$. The reason is that only in case of a high market demand the quantity of rooms

[27] For further elements about risk-neutrality and risk-aversion, linear and concave utility functions and expected utility maximizers with a Bernoulli utility function, see Mas-Colell A., Whinston M.D. and Green J.R. (1995).

[28] Because of the concavity of a risk-averse agent utility function, we assume that $u^{'} > 0$ and $u^{''} \leq 0$. Furthermore we assume that $u_o^{'} < 0$ and $u_E^{'} > 0$.

[29] Similarly to the "positive profit" condition, a "positive expected profit" condition for the Tour Operator implies that $\left(1-\alpha^H\right) u_{TO}\left[mD^L - oE\right] + \alpha^H u_{TO}\left[mE - oE\right] > 0$, condition as well respected if $m > o$.

[30] Because of the concavity of a risk-averse agent utility function, we assume that $u^{'} > 0$ and $u^{''} \leq 0$. Furthermore we assume that $u_o^{'} > 0$ and $u_E^{'} > 0$.

[31] We assume therefore the following Hotel variable production cost: $C(E) = cE$.

confirmed by the Tour Operator ($S = S^H$) can be not sufficient to satisfy entirely the market demand[32].

Therefore the Hotel utility becomes equal to:

$$\begin{aligned} U_h(E,o) &= \alpha^L u_h\left[\bar{P}S^L + (o-c)E\right] + \\ &\quad + \alpha^H u_h\left[\bar{P}S^H + (o-c)E + \left(\bar{P}-c\right)\left(D^H - S^H\right)\right] \end{aligned} \tag{6}$$

and applying the "short-side rule" (3) and taking into account that $\alpha^L = 1-\alpha^H$ we have:

$$U_h(E,o) = \left(1-\alpha^H\right) u_h\left[D^L + (o-c)E\right] + \alpha^H u_h\left[oE + (1-c)D^H\right] \tag{7}$$

The Hotel participation constraint takes instead the form $U_h(E,o) \geq \bar{U}_h$, where $\bar{U}_h$ is the Hotel reservation utility, which is an exogenous variable given by the expected profit from the sales on the spot market:

$$\bar{U}_h = Ex\left[u_h\left(\left(\bar{P}-c\right)\min\left(\bar{K},D\right)\right)\right] \tag{8}$$

Finally, we have therefore the following optimization problem, in which the Tour Operator goal is to achieve the maximum expected profit, subject to the Hotel participation and capacity constraints:

$$\begin{aligned} \underset{E,o}{Max}\ U_{TO}(E,o) &= Ex\left[u_{TO}\left(\Pi_{TO}(E,o)\right)\right] \\ \text{s.t. } U_h(E,o) &= Ex\left[u_h\left(\Pi_h(E,o)\right)\right] \geq \bar{U}_h \text{ and } E \leq \bar{K} \end{aligned} \tag{9}$$

where $U_h(E,d) \geq \bar{U}_h$ is the participation constraint and $E \leq \bar{K}$ is the capacity constraint.

Since we postpone the discussion about the capacity constraint $E \leq \bar{K}$ to the qualitative analysis of the solution and as a result of the assumption that $\lambda_1 > 0$[33], from the Kuhn-Tucker first order condition (FOC) we obtain:

$$\lambda_1^* = \frac{\left(1-\alpha^H\right)u_{TO}^{'L} + \alpha^H u_{TO}^{'H}}{\left(1-\alpha^H\right)u_h^{'L} + \alpha^H u_h^{'H}} = \frac{Ex\left(u_{TO}^{'}\right)}{Ex\left(u_h^{'}\right)} \tag{10}$$

[32] The Hotel can sell these further rooms on the market - in excess with respect to the rooms confirmed by the Tour Operator - directly to the tourists at the higher spot market price $\bar{P}$, for an amount equal to $\left(\bar{P}-c\right)\left(D^H - S^H\right)$. As we have previously seen, the Hotel could sell directly to the tourists also the rooms released by the Tour Operator but in this case the all market demand is satisfied by the Tour Operator and accordingly the Hotel can sell the rooms only to the Tour Operator.

[33] As far as the participation constraint is concerned, it is economically rational to consider only the hypothesis of strictly equal $U_h(E,o) = \bar{U}_h$ (and therefore $\lambda_1 > 0$) rather than the hypothesis of strictly greater $U_h(E,o) > \bar{U}_h$ (and therefore $\lambda_1 = 0$) because in this way we minimize the Tour Operator costs, Hotel participation being equal. Hence, we obtain a higher maximum expected profit, which implies a higher maximum utility.

where $u'_{TO} = \frac{\partial u_{TO}}{\partial \Pi_{TO}}$, $u'^{L}_{TO} = \frac{\partial u_{TO}}{\partial \Pi^{L}_{TO}}$, $u'^{H}_{TO} = \frac{\partial u_{TO}}{\partial \Pi^{H}_{TO}}$, $u'_h = \frac{\partial u_h}{\partial \Pi_h}$, $u'^{L}_h = \frac{\partial u_h}{\partial \Pi^{L}_h}$ and $u'^{H}_h = \frac{\partial u_h}{\partial \Pi^{H}_h}$.

From the second FOC we obtain, by plugging in $\lambda_1 = \lambda_1^*$, the following equation:

$$\frac{Ex\left(u'_{TO}\right)}{Ex\left(u'_h\right)} = \lambda_1^* = -\frac{o\left[Ex\left(u'_{TO}\right)\right] - m\alpha^H u'^{H}_{TO}}{o\left[Ex\left(u'_h\right)\right] - c\left(1-\alpha^H\right)u'^{L}_h} \tag{11}$$

Finally, the third FOC is:

$$\left(1-\alpha^H\right)u_h\left[\left(1-c\right)D^L + oE\right] + \alpha^H u_h\left[oE + \left(1-c\right)D^H\right] - \bar{U}_h = 0 \tag{12}$$

From the second and third FOCs, we obtain the "efficient contract" in terms of the two decision variables $E^*$ and $o^*$ (conditional on the exogenous parameters $D^L, D^H, \bar{U}_h, \alpha^H, m, c$), but this is not a Pareto optimal solution because the Tour Operator marginal rate of substitution is not equal to the Hotel marginal rate of substitution[34].

From the equation (11) and from the Pareto optimal condition, we obtain the necessary condition (but not sufficient) for a Pareto optimal contract:

$$MRS_{TO} = MRS_h = \frac{m}{c} \tag{13}$$

where $MRS_{TO} = \frac{\left(1-\alpha^H\right)u'^{L}_{TO}}{\alpha^H u'^{H}_{TO}}$ and $MRS_h = \frac{\left(1-\alpha^H\right)u'^{L}_h}{\alpha^H u'^{H}_h}$ are the Tour Operator and the Hotel marginal rates of substitution between their marginal utilities in case of low and high profit.

This condition is called "optimal risk sharing" rule because it represents the condition under which a socially efficient risk sharing takes place, as for the two firms it is optimal to reciprocally ensure themselves. This rule can be expressed as follows:

$$\frac{u'^{L}_{TO}}{u'^{H}_{TO}} = \frac{u'^{L}_h}{u'^{H}_h} = \frac{\alpha^H}{1-\alpha^H}\cdot\frac{m}{c} \tag{14}$$

From the equation (14) it is possible to analyze the case of absence of aggregate uncertainty in the tourism market:

$$\frac{u'^{L}_{TO}}{u'^{H}_{TO}} = \frac{u'^{L}_h}{u'^{H}_h} = 1 \tag{15}$$

In this case the "optimal risk sharing" rule implies for the two firms the choice to fully ensure themselves, such that there is indifference between the two

[34] Note that the condition for the Pareto optimality of a risk allocation between two risk-averse agents (in our model the Tour Operator and the Hotel) is the equivalence between their marginal rates of substitution: $MRS_{TO} = MRS_h$.

possible states of nature: since $u'^{L}_{TO} = u'^{H}_{TO}$ and $u'^{L}_{h} = u'^{H}_{h}$ then $\Pi^{L}_{TO} = \Pi^{H}_{TO}$ and $\Pi^{L}_{h} = \Pi^{H}_{h}$.

In our model, the condition under which for the two firms it is optimal to take out a reciprocal full insurance is the following one:

$$\alpha^{H} = \frac{c}{m+c} \tag{16}$$

where it can be shown that $\alpha^{H} = \frac{c}{m+c} \leq 1$ since $m > 0$ by assumption. Starting from this condition and recalling that $0 \leq \alpha^{H} \leq 1$, we have the following "qualitative" conclusions about the Allotment contract and the "optimal risk sharing" rule: (i) only if $\alpha^{H} = \frac{c}{m+c}$ then $\Pi^{L}_{TO} = \Pi^{H}_{TO}$ and $\Pi^{L}_{h} = \Pi^{H}_{h}$, there is absence of aggregate uncertainty in the market and it is therefore optimal for the Tour Operator to propose to the Hotel an Allotment contract with a reciprocal full coverage; (ii) only if $\alpha^{H} \neq \frac{c}{m+c}$ then $\Pi^{L}_{TO} \gtrless \Pi^{H}_{TO}$ and $\Pi^{L}_{h} \gtrless \Pi^{H}_{h}$, there is aggregate uncertainty in the market and it is therefore optimal for the Tour Operator to propose to the Hotel an Allotment contract with a reciprocal partial coverage. In particular, because of concavity of the utility functions: (i) only if $\alpha^{H} > \frac{c}{m+c}$ then $\Pi^{L}_{TO} < \Pi^{H}_{TO}$ and $\Pi^{L}_{h} < \Pi^{H}_{h}$; (ii) only if $\alpha^{H} < \frac{c}{m+c}$ then $\Pi^{L}_{TO} > \Pi^{H}_{TO}$ and $\Pi^{L}_{h} > \Pi^{H}_{h}$.

In conclusion, we have shown that the Allotment contract is an option contract which is not completely Pareto efficient: in fact, by the assumption of two risk-averse agents, the efficient contract can lead to Pareto suboptimality. This is because the "optimal risk sharing" rule (14) is verified only depending on the value of $\alpha^{H}$ and of the parameters $m$ and $c$, while in general there exist the condition only for a suboptimal (but efficient) contract[35].

# 5 The Free Sale Model

The different attitude of the two firms towards the risk is the most important difference between the Allotment and the Free Sale contracts. While the existence of the Allotment contract is justified by the assumption that both the Tour Operator and the Hotel are risk-averse agents, behind the Free Sale contract there exists the following basic assumption: the Tour Operator is a risk-neutral agent[36] while the Hotel is a risk-averse agent[37].

The first assumption can be justified because the Tour Operator often is a firm large enough to bear the risk without going bankrupt. The costs will be

[35] From the Kuhn-Tucker first order conditions we can make instead the following qualitative analysis about the capacity constraint $E \leq \bar{K}$: (i) if $\bar{K} > D^{H}$, the constraint is not binding; (ii) if $\bar{K} \geq E^{*} \Rightarrow E = E^{*}$, the constraint is not binding; (iii) if $\bar{K} < E^{*} \Rightarrow E = \bar{K}$, the constraint is binding and the Hotel can not entirely fulfill the contract proposed by the Tour Operator.

[36] Formally, this implies that the Tour Operator has a linear utility function, i.e. an utility function that exhibits a constant marginal utility.

[37] Formally, this implies that the Hotel has a concave utility function, i.e. an utility function that exhibits a diminishing marginal utility.

thereby widely spread over the company's customers, shareholders and others. The assumption is reasonable especially when the firm is widely held and, thus, the owners are capable to diversify the risk via the financial markets[38]. In fact, the occurrence of two parties one of which is risk-averse and the other is risk-neutral, is a necessary condition for an insurance market, but it is not a sufficient condition for a Pareto optimal contract in a framework of "choices under uncertainty"[39].

The other obvious difference between the Allotment and the Free Sale contracts consists in their different contractual clauses. According to the Free Sale contract the Hotel sells in advance rooms to the Tour Operator at the price: $P^c = \bar{P} - I(E)$, where $P^c$ is the "contractual price"[40], $I(E)$ is the discount (insurance premium) granted by the Hotel to the Tour Operator and $E$ is the Hotel supply, i.e. the "contractual quantity" (rooms booked in advance). In this way the Hotel insures itself against the risk of unsold rooms, granting in return the discount-premium $I(E)$ to the Tour Operator. In case the quantity $E$ of rooms booked in advance by the Tour Operator is not sufficient to satisfy the market demand, then the Hotel has the possibility to sell these further rooms directly to the tourists at the higher spot market price $\bar{P}$.

In terms of the Free Sale contract clauses, the discount $I(E)$ is equal to a percentage of the accommodation value (constant function): $I(E) = d$, where $0 \leq d \leq 1$ is a decision variable. The two decision variables of the Free Sale model are: $E$, number of rooms booked in advance by the Tour Operator (contractual quantity) and $d$, discount rate given by the Hotel to the Tour Operator (implicit contractual price).

Once the Tour Operator has booked the rooms, it will include them in the "package tours" and it will sell them at a future selling price (usually through the intermediation of a Travel Agency) $P^s = \bar{P}(1+m)$[41]. Once again we normalize the spot market price $\bar{P} \equiv 1$, so that the contractual prices become[42]: $P^c = 1 - d$ and $P^s = 1 + m$.

The Tour Operator "positive profit" condition $P^s > P^c$ is respected if $m > -d$ (always true because $d \geq 0$ by assumption). The Hotel "positive profit" condition $P^c > c$ is instead respected if $0 < c < 1$ (given that $d \geq 0$ by assumption).

Again we assume the condition (2) on the quantity variable $E$, because it

[38] For a definition of risk-aversion and risk-neutrality, see Cooter R. and Ulen T. (2004) and Faure M. and Goran S. (2003).

[39] See McKenna C.J. (1986) and Rossini G. (1994).

[40] To ensure the economic significance of the price $P^c$, we introduce the following positivity constraint (positive price condition): $P^c = \bar{P} - I(E) \geq 0$ and therefore $0 \leq I(E) \leq \bar{P}$.

[41] The Tour Operator does not charge the mark-up on the full cost $P^c$, but on the spot market price $\bar{P}$. This assumption is coherent with the assumption of a monopoly market for the Tour Operator and it implies the possibility for the monopolist to fully get the discount given by the Hotel without transferring the lower cost to the final consumer (through a lower selling price $P^s$).

[42] As previously stressed, the economic significance of the price $P^c$ implies the following positivity constraint: $P^c = 1 - d \geq 0$ and therefore $0 \leq d \leq 1$, always true by definition. The positivity of the price $P^s$ is instead implicit in the assumption $m > 0$.

would not be economically rational to book: (i) less than $D^L$ rooms ($E < D^L$), otherwise the Tour Operator would certainly loose favorable opportunities, for an amount equal to $P^s\left(D^L - E\right)$; (ii) more than $D^H$ rooms ($E > D^H$), otherwise the Tour Operator would have a certain loss for the Free Sale contracts in excess (unsold rooms), for an amount equal to $P^c\left(E - D^H\right)$.

We always assume that the Tour Operator sales $S$ are a random variable conditional on observed demand, according to the "short-side rule" (3). For the Tour Operator it is therefore profitable to sell every room on the market, within booked rooms, otherwise: (i) if $D = D^L$, by selling $S^L < D^L$ the Tour Operator would have an opportunity cost equal to $\bar{P}^s\left(D^L - S^L\right)$ with probability $\alpha^L$; (ii) if $D = D^H$, by selling $S^H < E$ the Tour Operator would have a certain loss for unsold rooms, equal to $P^c\left(E - S^H\right)$ with probability $\alpha^H$.

To summarize, the Free Sale model differs from the Allotment model only for the following assumptions: (i) the Tour Operator is risk-neutral and the Hotel is risk-averse; (ii) the form of a Free Sale contract is analogous to an insurance contract. We want to find the "optimal contract" in terms of: (i) rooms booked in advance by the Tour Operator ($E$, insurance level); (ii) discount given by the Hotel ($d$, insurance premium).

Given the assumption of a risk-neutral Tour Operator, we can define its linear utility function as[43]: $U_{TO}\left(E,d\right) = Ex\left[\Pi_{TO}\left(E,d\right)\right]$. Since the Tour Operator profit is: $\Pi_{TO}\left(E,d\right) = P^sS - P^cE$, its expected value is: $Ex\left[P^sS - P^cE\right] = \left(1+m\right)\left(S^L\alpha^L + S^H\alpha^H\right) - \left(E - dE\right)$.

According to the "short-side rule" (3) and taking into account that $\alpha^L = 1 - \alpha^H$, we have[44]:

$$U_{TO}\left(E,d\right) = \left(1+m\right)\left[D^L - D^L\alpha^H + E\alpha^H\right] - \left(E - dE\right) \tag{17}$$

Given the assumption of a risk-averse Hotel, its utility function over its profit can be defined as[45]: $U_h\left(E,d\right) = Ex\left[u_h\left(\Pi_h\left(E,d\right)\right)\right]$. Since the Hotel profit is: $\Pi_h\left(E,d\right) = \left(P^c - c\right)E$, where $c > 0$ is the Hotel unit production cost, the Hotel utility is[46]:

$$U_h\left(E,d\right) = \left(1-\alpha^H\right)u_h\left[\left(1-d-c\right)E\right] + \alpha^H u_h\left[-dE + D^H\left(1-c\right)\right] \tag{18}$$

[43] We assume that $u_d^{'} > 0$ and $u_E^{'} > 0$.

[44] Similarly to the "positive profit" condition, a "positive expected profit" condition for the Tour Operator implies that $\left(1+m\right)\left[D^L - D^L\alpha^H + E\alpha^H\right] - \left(E - dE\right) > 0$, condition as well respected if $m > -d$.

[45] Because of the concavity of a risk-averse agent utility function, we assume that $u^{'} > 0$ and $u^{''} \leq 0$. Furthermore we assume that $u_d^{'} < 0$ and $u_E^{'} > 0$.

[46] The Hotel profit takes a different form depending on the actual market demand. The reason, as we have already seen for the Allotment contract, is that only in case of a high market demand ($D = D^H$) the quantity of rooms booked in advance by the Tour Operator ($E$) can be not sufficient to satisfy entirely the market demand. The Hotel can therefore sell these further rooms on the market - in excess with respect to the contractual commitment of the Tour Operator - directly to the tourists at the higher spot market price $\bar{P}$, for an amount equal to $\left(\bar{P} - c\right)\left(D^H - E\right)$.

The Hotel participation constraint is $U_h(E,d) \geq \bar{U}_h$, where $\bar{U}_h$ is the Hotel reservation utility (8). Therefore the optimization problem becomes:

$$\begin{aligned} \underset{E,d}{Max}\ U_{TO}(E,d) &= Ex\left[\Pi_{TO}(E,d)\right] \\ \text{s.t. } U_h(E,d) &= Ex\left[u_h\left(\Pi_h(E,d)\right)\right] \geq \bar{U}_h \text{ and } E \leq \bar{K} \end{aligned} \tag{19}$$

where $U_h(E,d) \geq \bar{U}_h$ is the participation constraint and $E \leq \bar{K}$ is the capacity constraint.

Since we postpone the discussion about the capacity constraint $E \leq \bar{K}$ to the qualitative analysis of the solution of the model, from the first FOC we obtain:

$$\lambda_1^* = \frac{1}{(1-\alpha^H)\,u_h'^L + \alpha^H u_h'^H} = \frac{1}{Ex\left(u_h'\right)} \tag{20}$$

From the second FOC we obtain, by plugging in $\lambda_1 = \lambda_1^*$, the following equation:

$$MRS_h = \frac{1-(1+m)\,\alpha^H}{(1+m)\,\alpha^H - c} \tag{21}$$

where $MRS_h = \frac{\left(1-\alpha^H\right)u_h'^L}{\alpha^H u_h'^H}$ is the Hotel marginal rate of substitution between the Hotel marginal utilities in case of low and high profit.

Finally, the third FOC is:

$$\left(1-\alpha^H\right)u_h\left[(1-d-c)\,E\right] + \alpha^H u_h\left[-dE + D^H(1-c)\right] - \bar{U}_h = 0 \tag{22}$$

From the second and third FOCs, we obtain the "efficient contract" in terms of the two decision variables $E^*$ and $d^*$ (conditional on the exogenous parameters $D^H, \bar{U}_h, \alpha^H, m, c$), but this is not a Pareto optimal solution because the Hotel marginal rate of substitution (21) is not equal to one[47].

From the equation (21) and from the Pareto optimal condition, we obtain the necessary condition (but not sufficient) for a Pareto optimal contract:

$$MRS_h = 1 \tag{23}$$

that can be expressed as follows:

$$\frac{u_h'^L}{u_h'^H} = \frac{\alpha^H}{1-\alpha^H} \cdot \frac{1-(1+m)\,\alpha^H}{(1+m)\,\alpha^H - c} = 1 \tag{24}$$

[47] Note that the condition for the Pareto optimality of a risk allocation between a risk-neutral agent (in our model the Tour Operator) and a risk-averse agent (in our model the Hotel) is the following one: $MRS_h = \frac{u_h'^L}{u_h'^H} = 1$.

This condition is called the "optimal risk allocation" rule, i.e. it is the condition under which a socially efficient risk allocation takes place, as for the Tour Operator it is optimal to fully ensure the Hotel[48]. This rule can be expressed as follows:

$$\alpha^H = \frac{c}{m+c} \tag{25}$$

The "optimal risk allocation" rule implies therefore that there is indifference between the two possible states of nature and the Hotel profits become deterministic: since $u_h^{'L} = u_h^{'H}$ then $\Pi_h^L = \Pi_h^H$.

Similarly to the Allotment model, in the Free Sale model the condition under which for the Hotel it is optimal to take out a full insurance is therefore the (25) where it can be shown that $\alpha^H = \frac{c}{m+c} \leq 1$ since $m > 0$ by assumption. Starting from this condition and recalling that $0 \leq \alpha^H \leq 1$, we have the following "qualitative" conclusions about the Free Sale contract: (i) full insurance only if $\alpha^H = \frac{c}{m+c}$ (then $\Pi_h^L = \Pi_h^H$); (ii) partial insurance only if $\alpha^H < \frac{c}{m+c}$ (then $\Pi_h^L < \Pi_h^H$); (iii) no insurance only if $\alpha^H > \frac{c}{m+c}$ (then $\Pi_h^L > \Pi_h^H$).

# 6 Conclusions

In this paper our intent was to model the Allotment and the Free Sale contracts, to carry out a comparative analysis between them in terms of their efficiency and Pareto optimality.

From the analytical solution of the two models we have shown that the exchange and the level of the two main contractual variables, the price ($o$ for the Allotment contract and $d$ for the Free Sale contract) and the number of booked rooms ($E$ for both the contracts), depend mainly on three factors: (i) the attitude towards the risk of the two agents; (ii) the probability they assign to the different possible stochastic events (high or low market demand); (iii) their bargaining power.

We have seen that the two contracts are coherent with the main stylized facts observable in the tourism market: the Allotment contract takes the form of an option contract (risk-sharing agreement) between two risk-averse agents, while the Free Sale contract takes the form of an insurance contract (risk-transfer transaction) between a risk-neutral agent and a risk-averse agent.

Through a comparative analysis in terms of efficiency of the contracts, we can affirm that both the contracts are not completely Pareto efficient because the solutions of the models are individually efficient (i.e. we have an "efficient contract") but not socially efficient or Pareto optimal (i.e. we do not have an "optimal contract"). In fact, the general solutions of the two models do not satisfy the corresponding conditions for the Pareto optimality in a framework of risk allocation between two risk-averse agents (for the Allotment contract) or

[48] In this way we have a Pareto optimal risk allocation because the risk is totally shifted to the least risk-averse transactor or to the low-cost risk bearer.

between a risk-neutral agent and a risk-averse agent (for the Free Sale contract), therefore they lead in both cases to a suboptimal (but efficient for the agents) contract.

Nevertheless, the Pareto optimal solutions - see the "optimal risk sharing" rule (13) and the "optimal risk allocation" rule (23) - depend: (i) on the probability $\alpha^H$ (or, analogously, $\alpha^L = 1-\alpha^H$) assigned to the future market demand; (ii) on the value of the exogenous parameters $m$ (Tour Operator mark-up) and $c$ (Hotel unit cost). In fact, for both the contracts the Pareto optimal condition for a full insurance rather than a partial insurance between the two parties, is the same: $\alpha^H \gtreqless \frac{c}{m+c}$.

In particular only if $\alpha^H = \frac{c}{m+c}$, then: (i) $MRS_{TO} = MRS_h = 1$ and the Allotment contract is optimal (therefore both for the Tour Operator and the Hotel it is optimal to fully ensure themselves); (ii) $MRS_h = 1$ and the Free Sale contract is optimal (therefore for the Tour Operator it is optimal to fully ensure the Hotel). On the contrary, only if $\alpha^H \neq \frac{c}{m+c}$, then for both contracts there are the conditions for the optimality of a partial insurance (only in the case of the Free Sale contract if $\alpha^H > \frac{c}{m+c}$ the contract is not profitable any longer for the Tour Operator). This is an important result, because it shows that the two contracts have a common threshold value (16 and 25) independently from the specification of the utility functions of the two agents (and therefore from their different attitude towards the risk) and independently from the different legal characteristics and nature of the contract.

There are several potential causes for which the contracts are not completely Pareto efficient. In both cases the reason of this Pareto suboptimality could be found in the legal specification of the contracts (i.e. in their terms and clauses), that could not represent a correct incentive towards an optimal behavior of the parties, or in the economic specification of the models. For the Allotment contract one possible cause could be the specification of the option price $O(E)$, while for the Free Sale it could be the specification of the discount rate $I(E)$. Moreover, the inefficiency may result from the assumption that two different objectives are mixed within the Allotment and the Free Sale contracts, i.e. profit maximization and risk allocation. A consequence of this argumentation is that it may be possible to modify the contracts in order to limit (or to remove) their inefficiencies[49].

Finally, it could be also interesting to analyze the effect on the optimal contractual choice (in particular on the choice between full and partial insurance) of different degrees of risk-aversion of the parties[50] or of different trends of the Tour Operator mark-up[51]. Moreover, it could be interesting to model the case in which the Tour Operator and the Hotel make a contract which is

[49] One possible revision could be, for instance, to change the specification of the option price $O(E)$ or of the discount rate $I(E)$, such that they are function both of the booked rooms $E$ and of the probability $\alpha^H$, i.e. $O(E, \alpha^H)$ and $I(E, \alpha^H)$.

[50] A big Hotel, for example, could be a firm more capable to bear the risk than a little Hotel.

[51] In fact, usually regarding the mark-up we observe a procyclical trend.

a mix between the Free Sale and the Allotment contractual clauses. Another assumption that could mitigate the inefficiencies of the two contracts may be the possibility for the Tour Operator to negotiate with more than one Hotel, i.e. to book rooms from different Hotels rather than from only one[52]. This strategy would allow to the Tour Operator both a risk diversification and to get rid of the single Hotel capacity constraint.

However, some other possible extensions of the model may be: introducing a certain degree of competition among the Tour Operators[53], assuming a simultaneous uncertainty in prices (market price) and quantities (market demand)[54], analyzing the possibility for the Tour Operator to follow a strategy of vertical integration[55] or applying the "mean-variance model"[56].

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[52] In this way, we would relax the assumption of a monopoly market for the Hotels.

[53] By relaxing the assumption of a monopoly market for the Tour Operators, the Hotel could look for alternative Tour Operators.

[54] This assumption implies for the two firms the exposition in the same time both to price risk and to demand risk.

[55] Through this strategy, the Tour Operator becomes the owner of the Hotel, with the purpose of reducing the number and the complexity of business relationships established with external firms.

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